A Simple Technique to Improve Performance of RAKE Receivers in CDMA PCS Indoor Systems

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Abstract - In channels where the delay spread is smaller than the chip interval, spread spectrum signals do not give rise to path diversity, thus a classical maximal ratio combiner (MRC)-RAKE receiver sets to unity the number of branches. In this situation, MRC-RAKE receivers with resolution reduction (RR) of the diversity branches may be used to significantly improve performance in the forward link of DS-CDMA systems [1]. As a result of the use of RR techniques, the noise components at the finger outputs are correlated. It has been shown that the MRC is suboptimal in the presence of correlated noise components. In this paper, a minimum mean squared error combiner (MMSEC) RAKE receiver with RR is proposed. The performance of the RAKE receiver with two branches is studied by means of theoretical analysis and computer simulations. To allow a simple practical implementation, a suboptimal structure of the MMSEC is also proposed. Frame Error Rate (FER) is obtained by using a simulation system based on the forward link of the digital cellular standard IS-95 in a typical indoor radio channel. Our results show that this new receiver scheme can achieve a gain of 3.6 dB over a conventional MRC-RAKE receiver at an FER of 0.01, with small increase in complexity.

I. INTRODUCTION

The direct-sequence code division multiple access (DS-CDMA) IS-95 system [2] with 1.23 MHz bandwidth was originally designed for an outdoor cellular system where the delay spread is usually in the range of 10 μs. However, the delay spread for an indoor environment is generally around 100 ns, which cannot be resolved by a CDMA receiver with an 813 ns chip interval.

Resolution reduction is a method that improves the performance of CDMA systems in channels where the delay spread is smaller than the chip duration [1]. This technique consists in using the RAKE receiver with branches spaced less than one chip period apart. It has been show in [1] that this technique produces diversity gain in spite of the fact that the signals at the output of the RAKE are correlated. Several authors have analyzed the effects of the path signals correlation on the system performance [3][4][5]. For example, Ling [3] found the matched filter bound for M-correlated path channels. In [4], Al-Hussaini and Al-Basiouni studied the effect of the correlation on the performance of a dual-branch maximal ratio combiner (MRC), while Aalo [5] analyzed the performance of an M-branch MRC in a correlated Nakagami-fading channel. All these works show that even when the path signals are correlated, it is possible to obtain an important improvement with respect to the case of only one path. Based on this result, Yang [1] proposed a reduction of the interval between diversity branches in order to improve the performance of an MRC-RAKE receiver on indoor radio channels. In this case, a reduction to one half of the chip interval significantly improves the Frame Error Rate (FER) in the forward link of the IS-95 standard [2]. However, a detailed theoretical study of resolution reduction techniques has not been reported so far.

In this work, we first analyze the signal at the mobile station: the transmitted signal from its base station and the noise signal (thermal noise + interference from other cells). We show that due to the RR and the effect of the receiver filter, the noise signals at the output of the RAKE are correlated. In this situation, an MRC is not effective therefore we introduce an optimum minimum mean square error combiner (MMSEC)[6]. The performance of a dual-branch MMSEC-RAKE receiver with RR in a typical indoor radio channel is investigated. By means of theoretical analysis and computer simulation we show that MMSEC outperforms MRC. Our study considers the chip pulse shape used in practical implementations and the effects of uncorrelated noise components present at the RAKE input, which are not affected by the RR (e.g., quantization noise from A/D converters). To allow a simple practical implementation, a suboptimal structure of the MMSEC is also proposed. Frame Error Rate improvement is demonstrated by using a simulation system based on the forward link of the IS-95 standard.

The paper is organized as follows. In Section 2 we derive expressions for the signal statistics at the output of the RAKE receiver. The optimum combiner (MMSEC) is presented and analyzed in Section 3. Computer simulation and discussions are contained in Section 4. Finally, concluding remarks are given in Section 5.

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II. THE RECEIVED SIGNAL AT THE MOBILE

In this section we consider the signal received at the mobile terminal (specifically, in this paper we concentrate on the forward link of the IS-95 standard [2]).

A. The Channel Model

The impulse response of a frequency selective multipath radio channel at time $t$ can be represented by

$$h(t) = \sum_{l=0}^{L-1} \beta_l(t) \delta(t - \tau_l),$$

where $L$ is the total number of multipaths, $\beta_l(t)$ and $\tau_l$ are, respectively, the complex amplitude and the delay of the $l$th multipath. Note that, for a Rayleigh fading channel, $\beta_l(t)$ is a zero-mean complex Gaussian random process. The channel used in this paper is shown in Table I [1]. Note that the mean delay spread of the channel (which will be denoted by $d_{\text{spread}}$) is around 100 ns.

B. DS-CDMA Receiver

For a slow fading channel, the equivalent lowpass received signal can be represented as (we assume perfect carrier recovery)

$$r(t) = \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \beta_l(t) b_k[i] N \alpha_k \phi(t - i T_r - \tau_l)$$

$$+ z(t),$$

where $K$ is the number of users ($k = 0$ denotes the pilot signal), $T_r$ is the chip period (e.g., $T_r = 813$ ns for IS-95), $\alpha_k$ and $A_k$ are the voice activity factor and the signal amplitude, respectively; $\phi(t) = \frac{\sin(\pi t / T_r)}{\sqrt{T_r}}$ is the impulse response of the transmitter (receiver) filter, which corresponds closely to the chip pulse shape used in the IS-95 standard [2]. $\alpha_k[i] = (\pm 1 + j) / \sqrt{2}, j = \sqrt{-1}$ is the complex chip sequence for the $k^\text{th}$ user; we assume that the different chip sequences are orthogonal because Walsh functions are used in the code sequence. $b_k[i] = \sum_m b_k[m] p_N[i - m N]$ is the information sequence, where $b_k[m] = \pm 1$ is the data symbol, $N$ is the processing gain and $p_N[i] = 1$ if $i = 0 \ldots N - 1$; $p_N[i] = 0$ elsewhere. $z(t)$ represents the complex noise (thermal noise + interference from other cells), which is modeled as zero-mean AWGN with $E\{z(t) z^*(t)\} = \sigma_n^2 \delta(t)$. ($E\{\cdot\}$ denotes the expectation operator and the superscript * indicates complex conjugation).

At the receiver, $r(t)$ is filtered and sampled each $T_r / M$ seconds (e.g., $M = 2, 4, 8$). Then, the samples are processed by synchronization stages (code acquisition, alignment and tracking) and by the RAKE fingers (despreaders) [7][8]. After the searching process, the output of the $j^\text{th}$ finger at the $n^\text{th}$ instant for the $k^\text{th}$ user is

$$y_j[m] = \frac{1}{N} \sum_{n=mN+n_j}^{N+n_j} a_k[n - n_j] r_j[n T_r + \Delta \tau_j] + \mu_j[n].$$

(3)

where $r_j(t) = r(t) * \phi(t)$ is the signal at the receiver filter output (the operator * denotes convolution), $a_k[n]$ is the chip sequence of the $k^\text{th}$ user, $n T_r + \Delta \tau_j$ is the sampling instant and $\Delta \tau_j = q_j T_r / M$ with $q_j = 0, 1, 2, \ldots, M - 1$ is the sampling phase. Note that

$$n_j = n_j T_r + \Delta \tau_j.$$  

(4)

with $n_j = 0, 1, 2, \ldots, N - 1$, is the delay corresponding to the $j^\text{th}$ finger. The proper values of $n_j$ and $q_j$ are determined by synchronization stages. We have included $\mu_j[n]$ in our model to consider the effects of uncorrelated noise components present after the receiver filter (e.g., quantization noise from A/D converters). We will refer to this component as "internal noise". In this work we assume that $\mu_j[n]$ is zero-mean AWGN with $E\{\mu_j[n] \mu_k^*[n]\} = \delta_{mn} \delta_{jk} \mu_j$. ($\delta_{mn} = 1$ if $m = n$, $\delta_{mn} = 0$ if $m \neq n$).

Equation (3) can be expressed in the following compact form

$$y_j[m] = b_k[m] s_j[m] + \eta_j[m] + \xi_j[m] + \chi_j[m].$$

(5)

Next we analyze each one of the components of (5):

1. $b_k[m] s_j[m]$: desired signal component for the $k^\text{th}$ user. For a slow fading channel, $s_j[m]$ is given by
\[ s_j[m] = \alpha_j A_j \sum_{i=0}^{J-1} \beta_i[m] \Theta(\tau_{ji}), \]  
(6)

where \( \Theta(\tau_{ji}) = \phi(\tau) \ast \phi(\tau) = \sin(\frac{\tau_{ji}}{T_s}) \) and \( \tau_{ji} = \tau_j - \tau_i \) and \( \beta_i[m] = \beta_i(mN + n_j)T_s + \Delta \tau_j \) (\( \beta_i(t) \) is assumed constant in a time interval of \( NT_s \) [9]). From (6) it can be seen that \( s_j[m] \) results in a zero-mean complex Gaussian variable. Note also that the different multipaths interfere among themselves because \( |\tau_j - \tau_i| < T_s \), \( \forall i \neq j \).

2. \( \chi_j[m] \): interference from the same cell-site users due to the presence of the multipaths and imperfect chip timing. Since that in our case \( d_{\text{spread}} < T_c \), we have verified that the effects of \( \chi_j[m] \) are negligible (this study is not included here). Moreover, since the fading is much slower than symbol rate, it is easy to cancel the pilot interference prior to combining, which is about 20% of the total downlink power [10]. For these reasons, in this work we do not consider these interference components.

3. \( \eta_j[m] \): interference due to the AWGN generated from thermal noise and the noise from adjacent cell-site base stations. When \( N \gg 1 \), \( a_{\text{ch}}[n] \) can be treated as a coin-flipping sequence [11], thus we can assume that \( a_{\text{ch}}[n] \) is a zero-mean random process with \( \mathbb{E}[a_{\text{ch}}[n]|a_{\text{ch}}[n] = \delta_{nm} \). Then, using the central limit theorem, it can be shown that \( \eta_j[m] \) results in a zero-mean Gaussian process, with

\[ \mathbb{E}[\{s_j[m]|n_k[r]\}] = \delta_{nm} \mathbb{E}[\Theta(\tau_{ji} - \tau_{kj})]. \]  
(7)

4. \( \xi_j[m] \): interference from the uncorrelated noise components \( \mu_j[m] \). Similar to the above case, we may be show that \( \xi_j[m] \) can be assumed as a zero-mean Gaussian process with \( \mathbb{E}[\{r_j[m]|r_k[m]\}] = \delta_{nm} \delta_{jk} \mathbb{E}[\Theta(\tau_{ji} - \tau_{kj})]. \) Note also that \( \eta_j[m] \) and \( \xi_j[m] \) are uncorrelated.

III. RAKE RECEIVER WITH MINIMUM MEAN SQUARE ERROR COMBINER (MMSEC)

Path diversity is not available when \( d_{\text{spread}} \) is smaller than \( T_c \). Similar to [1], we decrease the resolution between diversity branches with the objective of obtaining diversity gain (i.e., \( |\tau_j - \tau_i| < T_c \), \( j \neq k \)). We consider a RAKE receiver with two branches. In this case, the second finger is used to demodulate the received signal with a delay \( T_1 \), respect to the main finger, less than \( T_c \) (that is, \( |T_1 - \tau_0| < T_c \)).

The RAKE output for the \( k_1 \) user can be expressed as

\[ Y[m] = b_{k_1}[m] S[m] + Z[m], \]  
(8)

where \( Y[m] = [y_0[m] y_1[m]]^T \), \( S[m] = [s_0[m] s_1[m]]^T \).

\[ Z[m] = [\eta_0[m] + \xi_0[m] \eta_1[m] + \xi_1[m]]^T \] (the superscript \( \dagger \) denotes transposition). Note that for a given set of fade values, \( Y[m] \) is a Gaussian vector.

Since \( |t_1| < T_c \), the noise correlation factor is different from zero (see (7)), and the MRC results suboptimum. In this case it is well-known that the optimum combiner is the MMSEC, which is given by [6]

\[ W_{\text{opt}} = S^T R^{-1}, \]  
(9)

where \( R^{-1} \) is the inverse of the autocovariance matrix of the noise components at the finger outputs, which is defined by \( R = E[ZZ^T] \) (the superscript \( \dagger \) indicates conjugate transpose).

A. Performance of the MMSEC-RAKE Receiver

Using (8) and (9), the output of the MMSEC can be expressed as a simple decision variable in the form [12]

\[ u_r = \text{Re}\{S^T R^{-1} S + S^T R^{-1} Z\}. \]  
(10)

For a given set of fade values, \( u_r \) is a Gaussian variable with mean \( S^T R^{-1} S \) and variance \( \sigma_r^2 = 0.5 S^T R^{-1} S \). Then, it is possible to verify that the average bit error rate is [3] [12] [13]

\[ P_e = 0.5 \sum_{j=0}^{1} p_j \left[1 - \sqrt{\frac{\delta_j}{(1 + \delta_j)}}\right], \]  
(11)

where \( p_j = \delta_j/(\delta_j - \delta_k) \) \( j,k = 0,1 \) with \( j \neq k \). Components \( \delta_j \) are the eigenvalues of the matrix \( D \) given by

\[ D = \Lambda^{1/2} P^T R^{-1} P \Lambda^{1/2}, \]  
(12)

where \( P \) is the matrix of eigenvectors of \( C_z = E[SS^T] \) and \( \Lambda \) is the diagonal matrix of real, positive eigenvalues of \( C_z \). For example, if \( E[|x|^2] = E[|s_0|^2] \), the bit error probability of the MMSEC is given by (11) with
where $\rho_{\text{sig}}$ and $\rho_{\text{noise}}$ are the correlation factors of the signal and noise components, respectively.

IV. SIMULATION RESULTS AND DISCUSSION

Next, the performance of the MRC and MMSEC for different values of $t_1$ is investigated by means of computer simulations. The channel of Table I is used. The output of the RAKE is given by (8). $\eta_{0}[m]$ and $\eta_{1}[m]$ are zero-mean AWGN with variances $I_{\text{in}}/N$. The correlation factor between these components is $\Theta(t_1)$ (see (7)). $\zeta_{0}[m]$ and $\zeta_{1}[m]$ consist of zero-mean AWGN with variances $I_{\text{in}}/N$ (note that these components are uncorrelated). We have used $I_{\text{in}}/I_{\text{un}} = 15dB$. Note that since $d_{\text{spread}} < T_c$, a classical MRC-RAKE receiver sets to unity the number of branches ($t_1=0$).

The signal-to-noise ratio at the main branch ($SNR_{\text{MB}}$), used in the results presented in this paper, is defined by:

$$SNR_{\text{MB}} = \frac{N|\xi_0|^2}{I_{\text{in}} + I_{\text{un}}}.$$  \hspace{1cm} (14)

Fig. 1 shows the average signal-to-noise ratio at the main branch ($SNR_{\text{MB}}$) for different values of $t_1$, required to achieve a bit-error-rate (BER) of $10^{-2}$ and $10^{-3}$. In this figure is also shown the theoretical results predicted by equation (11) (solid line). Note the extremely close agreement between theory and simulation. To evidence the effects of $\zeta_{0}[m]$ we include the performance of the MMSEC for $I_{\text{un}} = 0$ (i.e., without uncorrelated noise components).

As can be seen, for $t_1 \to 0$ the performance of the MRC is similar to the case of only one path (gain diversity is not present). Note that when $t_1$ increases, we have that:

a) the correlation factor between signal components decreases $\Rightarrow$ it gives rise to diversity gain;

b) the correlation factor between noise components decreases $\Rightarrow$ MRC tends towards the optimal operation (uncorrelated noise);

c) the average power on the second finger is reduced.

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2 For $\rho_{\text{noise}}=0$, eq (11) is similar to the expression found by Ling [3]. Note also that if $\rho_{\text{noise}}=\rho_{\text{sig}}$ (both smaller than one), the performance of the MMSEC is similar to a MRC with two independent paths.

3 Note that for $t_1<<T_c$, $E[|\xi_0|^2] = E[|\xi_1|^2]$. Then, starting with zero, the behavior of the MRC improves when $t_1$ grows, until a minimum signal-to-noise ratio is reached. This improvement occurs because the effects a) and b) are dominant. Continuing to increase the value of $t_1$, the performance of the MRC is deteriorated due to the power reduction on the second finger (effect c).

It can be seen that the performance of the MMSEC is similar to the MRC with one finger when $t_1 \to 0$. This result can be obtained from (13) since $I_{\text{un}}/I_{\text{in}} = 0$ when $t_1 \to 0$ due to the presence of the uncorrelated noise components $\zeta_{0}[m]$. As $t_1$ is increased, the performance improves respect to MRC. This is because the MMSEC can perform a better cancellation of the noise which, according with (13), tends to enhance the diversity gain from the correlated signal components $\xi_0$ and $\xi_1$. Finally, when $t_1 \to T_c$ the performances of the MMSEC and MRC are the same (see effect c).

It can be seen that for the MMSEC a gain of about 3.0 dB from a MRC-RAKE receiver with one branch is obtained at BER $= 10^{-2}$. The corresponding gain at $BER = 10^{-3}$ is about 6.40 dB. Note also that the gain of the MMSEC over a classical MRC with two fingers ($t_1 = T_c$) is not so large. However, for $t_1 = T_c$, the signal power of the second finger is very small respect to the main finger ($=17dB$ in our case), which may be problematic in practical implementations. This difficulty may be minimized with the MMSEC using, for example, $t_1 = T_c/4$ or $T_c/2$ (note that in these cases MMSEC outperforms MRC).
From Fig. 1 note that the effects of the uncorrelated noise components are not so important for $t_i > 0.25T_e$ (compare curves “o” and “+”). Thus, we can use the following suboptimal combiner coefficients

$$\mathbf{W}_s^T = \mathbf{S}^T \mathbf{R}_s^{-1},$$

(15)
with

$$\mathbf{R}_s = \begin{bmatrix} 1 & \Theta(t_i) \\ \Theta(t_i) & 1 \end{bmatrix},$$

which are obtained from (9) ignoring the internal noise components. This suboptimal structure of the MMSEC is simple to implement because it does not require any additional estimation compared with an MRC (for a given value of $t_i$ and receiver filter, $\Theta(t_i)$ is known).

Next, we simulate the proposed receiver using a rate $\frac{1}{2}$, constraint length 9 convolutional code with interleaving as specified in the IS-95 standard and soft-decision decoding. Sample rate is eight times the chip rate ($M=8$), data rate is set to full voice rate 9600 bps ($\alpha_{k_i} = 1$), symbol rate of $b_{k_i}(m)$ is 19200 symbols/s, carrier frequency is 1800 MHz and Doppler frequency is $f_d = 8.33$ Hz. Frame Error Rate (FER) for different values of the signal level is shown in Fig. 2. FER was obtained using the MMSEC with $t_i = T_e / 2$. Since we are using a rate $\frac{1}{2}$ code, the signal to noise ratio is defined by $E_b / N_0 = 2SNR_{dB}$. Note that a gain of about 3.6 dB and 1.2 dB over a MRC-RAKE receiver with one and two fingers ($t_i = T_e / 2$) respectively are obtained at a FER=0.01. It is important to realize that these gains are achieved with small increase in complexity.

V. CONCLUDING REMARKS

By means of theoretical analysis and computer simulations, the use of the RAKE receivers with reduction of the interval between diversity branches has been demonstrated to be a viable alternative to increase the capacity of an indoor DS-CDMA system when the bandwidth of the spread spectrum signal is not sufficiently large to give rise to path diversity. We have shown that the behavior of the RAKE receiver with RR in a typical indoor radio channel can be substantially improved when an MMSEC is used instead of an MRC. It was also noted that a reduction of the resolution to less than about 0.25$T_e$ may be not advantageous due to real world limitations. Moreover, to allow a simple practical implementation, a suboptimal structure of the MMSEC was proposed (compared with an MRC, only a pair of additional operations per combiner coefficient are necessary). Simulations of this scheme in the forward link of the IS-95 standard, demonstrated that a gain of about 3.6 dB over a classical MRC-RAKE receiver without RR can be obtained at FER of 0.01.

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